Entrepreneurial Income Inequality, Aggregate Saving and the Gains from Trade

Online Appendix

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Contents

E The Impact of International Trade on Inequality: Analytical Results

I first examine the effects of trade on inequality. Although the model is dynamic in nature, the production decisions of entrepreneurs are static. This allows us to derive some analytical results.

Proposition 1. Moving from Autarky ($\tau = \infty$) to any positive level of trade ($\tau < \infty$, $e(z) = 1$ for some z), the profit share of the top x% of entrepreneurs strictly increases for any $x \in (0, 100)$, for any non-degenerate CDF function $\mu(z)$.

Lemma 1. If $f(x) > 0 \; \forall x \in (z, \infty)$, and $\sigma > 1$, then

$$\frac{d\left(\frac{\int_z^\infty x^{\sigma-1} f(x)dx}{\int_z^\infty f(x)dx}\right)}{dz} > 0.$$

Proof:

$$\begin{aligned}
\frac{d\left(\frac{\int_z^\infty x^{\sigma-1} f(x)dx}{\int_z^\infty f(x)dx}\right)}{dz} &= \frac{\int_z^\infty f(x)dx \cdot \Big(-z^{\sigma-1} f(z)\Big) - \int_z^\infty x^{\sigma-1} f(x)dx \cdot (-f(z))}{(\int_z^\infty f(x)dx)^2} \\
&= \frac{f(z) \cdot \int_z^\infty (x^{\sigma-1} - z^{\sigma\text{-}1}) f(x)dx}{(\int_z^\infty f(x)dx)^2} \\
&> 0
\end{aligned}$$

Proof of Proposition 1**.**

Define z_x as $\mu(z_x) = 1 - \frac{x}{100}$. In Autarky, the profit share of the top x% of entrepreneurs is given by

$$\frac{\int_{z_x}^\infty z^{\sigma-1}\pi^D(z_{min})\mu(dz)}{\int_{z_m}^\infty z^{\sigma-1}\pi^D(z_{min})\mu(dz)} = \frac{\int_{z_x}^\infty z^{\sigma-1}\mu(dz)}{\int_{z_m}^\infty z^{\sigma-1}\mu(dz)}$$

where $\pi^D(.)$ is defined earlier. Constant-returns-to-scale (CRS) production and the demand function imply that $\frac{\pi^D(z)}{\pi^D(z_{min})} = \frac{z^{\sigma-1}}{z_{min}^{\sigma-1}}$. The profit from exporting activities for a firm with productivity z is given by $\tau^{1-\sigma}\pi^D(z) - w \cdot f_X$. Recall that $\bar{z}_X$ is the productivity cutoff for exporting.

Case 1: $z_x > \bar{z}_X$**.**

By Lemma (1),

$$\frac{\int_{z_x}^\infty z^{\sigma-1}\mu(dx)}{\int_{z_x}^\infty \mu(dx)} > \frac{\int_{\bar{z}_X}^\infty z^{\sigma-1}\mu(dx)}{\int_{\bar{z}_X}^\infty \mu(dx)}$$

which implies

$$\frac{\frac{w \cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{z_x}^\infty \mu(dx)}{\frac{w \cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{\bar{z}_X}^\infty \mu(dx)} < \frac{\int_{z_x}^\infty z^{\sigma-1}\mu(dx)}{\int_{\bar{z}_X}^\infty z^{\sigma-1}\mu(dx)},$$

where $\pi_T^D(.)$ is the domestic profit function. Since exporters export only if export revenue is greater than the fixed cost of exporting, $\tau^{1-\sigma}\pi_T^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) > w \cdot f_X \int_{z_x}^{\infty} \mu(dx)$.

It can shown that if $\frac{A}{B} > \frac{C}{D}$, and $B > D > 0$, then $\frac{A-C}{B-D} > \frac{A}{B}$. This implies

$$\frac{\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) - \frac{w\cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{z_x}^{\infty}\mu(dx)}{\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - \frac{w\cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{\bar{z}_X}^{\infty}\mu(dx)} > \frac{\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx)}{\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx)}$$

Since $\bar{z}_X > z_{min}$,

$$\frac{\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) - \frac{w\cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{z_x}^{\infty}\mu(dx)}{\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - \frac{w\cdot f_X}{\pi_T^D(z_{min})}\tau^{\sigma-1}\int_{\bar{z}_X}^{\infty}\mu(dx)} > \frac{\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx)}{\int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx)}$$

It can be shown that if $\frac{A}{B} > \frac{C}{D}$, $B > 0$, and $D > 0$, then $\frac{A+C}{B+D} > \frac{C}{D}$. I obtain

$$\frac{\pi_T^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) + \tau^{1-\sigma}\pi_T^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) - w\cdot f_X\int_{z_x}^{\infty}\mu(dx)}{\pi_T^D(z_{min})\int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx) + \tau^{1-\sigma}\pi_T^D(z_{min})\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - w\cdot f_X\int_{\bar{z}_X}^{\infty}\mu(dx)} > \frac{\pi_A^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx)}{\pi_A^D(z_{min})\int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx)},$$

where $\pi_A^D(.)$ is the profit function of a firm in Autarky. The left-hand side of the above equation gives the share of total profit received by the top x% of firms in Trade, while the right-hand side gives the corresponding share in Autarky.

Case 2: $z_x \leq \bar{z}_X$**.**

Since exporters export only if export revenue is greater than fixed cost of exporting,

$$\tau^{1-\sigma}\pi_T^D(z_{min})\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - w\cdot f_X\int_{\bar{z}_X}^{\infty}\mu(dx) > 0$$

Furthermore, since $z_x > z_{min}$,$\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) < \int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx)$.

It can be shown that if $A < B$, $B > 0$ and $C > 0$, then $\frac{A}{B} > \frac{A+C}{B+C}$. This implies

$$\frac{\pi_T^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx) + \tau^{1-\sigma}\pi_T^D(z_{min})\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - w\cdot f_X\int_{\bar{z}_X}^{\infty}\mu(dx)}{\pi_T^D(z_{min})\int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx) + \tau^{1-\sigma}\pi_T^D(z_{min})\int_{\bar{z}_X}^{\infty} z^{\sigma-1}\mu(dx) - w\cdot f_X\int_{\bar{z}_X}^{\infty}\mu(dx)} > \frac{\pi_A^D(z_{min})\int_{z_x}^{\infty} z^{\sigma-1}\mu(dx)}{\pi_A^D(z_{min})\int_{z_{min}}^{\infty} z^{\sigma-1}\mu(dx)}.$$

The left-hand side of the above equation gives the share of total profit received by the top x% of firms in Trade, while the right-hand side of the above equation gives the corresponding share in Autarky.

A corollary of Proposition 1 is that the Gini coefficient of profits among entrepreneurs is minimized at Autarky. It is important to emphasize that Proposition 1 considers only the distribution of profit income among entrepreneurs. The distribution of interest income is determined by dynamic factors such as the persistence of profit income and risk aversion, so it is difficult to examine analytically.

The effects of trade on the inequality between workers and entrepreneurs are summarized by the following proposition.

Proposition 2. Consider the special case of the model in which there is no capital depreciation ($\delta = 0$). Moving from Autarky ($\tau = \infty$) to any positive level of trade ($\tau < \infty$, $e(z) = 1$ for some z), the share of total income received by the workers increases.

Proof: The total wage bill in the economy is w while the total cost of capital rental before depreciation is RK. Recall that L_v denotes the total *variable* labor input. As is well-known, $\frac{w \cdot L_v}{r^K K} = \frac{1-\alpha}{\alpha}$ with Cobb-Douglas production. Therefore, with $\delta = 0$, $rk = r^K K = w \cdot L_v \cdot \frac{\alpha}{1-\alpha}$.

Denote the equilibrium wages in Autarky and under Trade as w_A and w_T. With the CES monopolistic framework, total entrepreneurial profit in Autarky is given by $\frac{\sigma}{\sigma-1}\frac{w_A \cdot L_v}{1-\alpha} = \frac{\sigma}{\sigma-1}\frac{w_A}{1-\alpha}$, where the second inequality follows because $L_v = 1$ under Autarky. Denoting the fraction of exporters under Trade as pctX, $0 < \text{pctX} < 1$, total entrepreneurial profit in Trade is given by $\frac{\sigma}{\sigma-1}\frac{w_T \cdot L_v^T}{1-\alpha} - \text{pctX} \cdot w_T \cdot f_X$, $L_v^T < 1$.

In Autarky, the share of total income received by workers is given by

$$\frac{w_A}{w_A + (\frac{w_A \cdot \alpha}{1-\alpha} + \frac{\sigma}{\sigma-1}\frac{w_A}{1-\alpha})} = (1-\alpha)\frac{\sigma-1}{2\sigma-1} \tag{E.1}$$

Analogously under Trade, the share of total income received by workers is given by

$$\begin{aligned} & \frac{w_T}{(L_v^T \cdot w_T + \text{pctX} \cdot w_T \cdot f_X) + (\frac{L_v^T \cdot w^T \cdot \alpha}{1-\alpha} + \frac{\sigma}{\sigma-1}\frac{L_v^T w_T}{1-\alpha} - \text{pctX} \cdot w_T \cdot f_X)} && \text{(E.2)} \\ = \quad & \frac{1}{L_v^T}(1-\alpha)\frac{\sigma-1}{2\sigma-1} && \text{(E.3)} \end{aligned}$$

The value in Equation (E.3) is lower than that in Equation (E.1) by inspection.

With constant mark-up and Cobb-Douglass production, total *variable* profit, total cost of capital rental and total cost of *variable* labor input are fixed in proportion across all firms, regardless of the level of trade cost. However, as more firms start to export, more labor is used to cover the fixed cost of exporting. As a result, total labor income increases relative to total entrepreneurial income. Also, since the markup of price over marginal cost is constant, the percentage markup of price over average cost is lower at exporting firms, as a result of the fixed cost of exporting. Therefore, compared to Autarky, total profit (net of the fixed cost of exporting) as a share of total sales is lower in an economy with any positive level of trade ($\tau < \infty$). Therefore, in moving from

Autarky ($\tau = \infty$) to any positive level of trade ($\tau < \infty$), the share of total income received by the workers increases.

F Decomposition of Saving Rates and Target Wealth to Income Ratio

F.1 A decomposition of the change in aggregate saving rate by entrepreneurs.

Proposition 3. In a stationary equilibrium, the aggregate saving rate of entrepreneurs is zero.

Proof: From the budget constraint of an entrepreneur, we have

$$c(a,z) + a'(a,z) = \max\{\pi^D(z), \pi^X(z)\} + (1+r)a.$$

We then integrate this budget constraint over all a and z,

$$\int_a \int_z c(a,z)dadz + \int_a \int_z a'(a,z)dadz = \int \max\{\pi^D(z), \pi^X(z)\}dz + \int ada + r\int ada.$$

Denoting the aggregate quantities with C,K',Π and K, respectively, we have

$$C + K' = \Pi + K + rK.$$

In a stationary equilibrium, we have $K = K'$. Therefore, we have $C = \Pi + rK$. The left hand side is consumption while the right hand side is total income, which consists of total profit income and total interest income. The aggregate saving rate of entrepreneurs is given by $SR_s = \frac{\Pi + rK - C}{\Pi + rK} = 0$, {s=A, T}. Therefore, when we compare the stationary equilibria under Autarky and under Trade, the aggregate saving rate by entrepreneurs is necessarily the same at 0.[1]

Let the average saving rate of entrepreneurs with productivity z be $sr_s(z) = \frac{y_s(z) - c_s(z)}{y_s(z)}$. Then the aggregate saving rate equals $SR = \frac{\int y_s(z) - c_s(z)\mu(dz)}{\int y_s(z)\mu(dz)} = \int sr_s(z) \cdot share_s^y(z)\mu(dz)$ where $y_s(z)$ is the average income of all entrepreneurs with productivity z,[2] and $share^y(z) = \frac{y_s(z)}{\int y_s(z)\mu(dz)}$. Follow-

[1] Moving from Autarky to Trade, the gross saving rate of the economy increases. The aggregate capital stock in the economy is K. In each period, δK amount of capital is depreciated, and the same amount must be saved to maintain the capital stock at K. Therefore, the gross saving rate of the economy in steady state is given by $\frac{\delta K}{Y}$. Since K increases more proportionally than Y from Autarky to Trade in the calibration exercise, the gross saving rate $\frac{\delta K}{Y}$ is higher under Trade than under Autarky. It is helpful to note that the replacement of depreciated capital is carried out by financial intermediaries instead of by entrepreneurs in the model, and entrepreneurs earn the net return of saving r (instead of "$R = r + \delta$").

[2] The average is taken over entrepreneurs with different a. I am grouping the entrepreneurs by z instead of by (a,z) because the joint distribution of (a,z) is an endogenous object. It is not possible to match the entrepreneurs by (a,z) between the equilibria under Autarky and under Trade.

ing Olley and Pakes (1996),

$$SR_T - SR_A = \int sr_A(z)\big(share^y_T(z) - share^y_A(z)\big)\mu(dz) + \int \big(sr_T(z) - sr_A(z)\big)share^y_A(z)\mu(dz) + \int \big(sr_T(z) - sr_A(z)\big)\big(share^y_T - share^y_A(z)\big)\mu(dz) \tag{F.4}$$

The first term in Equation (F.4) is the "between" change in the saving rate, which is the change in the saving rate if we hold the average saving rate of all entrepreneurs of a given z fixed at its level under Autarky, but change the income shares to their levels under Trade. The second term is the "within" change in the saving rate, which is the change in the saving rate if we fix the income share of all entrepreneurs at a given z at its level under Autarky, but change the saving rates to their levels under Trade. In the baseline calibration of the full model, I find the "between" change to be 1.72 percentage points and the "within" change to be -1.83 percentage points. The last term is a co-variance term relating changes in income shares to changes in the individual saving rate. The covariance term is 0.11 percentage point in the current application.

F.2 Decomposition based on the Target-Wealth-to-Profit Ratio

Recall that $a'_s(a,z)$,$s = \{A, T\}$ is the asset policy function of an entrepreneur with asset a and productivity z, where the subscript s is added to emphasize that the policy function is dependent on the trade regime, and A and T denote "Autarky" and "Trade" respectively. Define $a^*_s(z)$ to be such that $a'_s(a^*_s(z), z) = a^*_s(z)$. The target wealth $a^*_s(z)$ of an entrepreneur is the steady state asset holding if the entrepreneur were to receive the same z forever.[3] Let the profit of an entrepreneur with productivity z be $\pi_s(z)$, $s = \{A, T\}$. The target-wealth-to-profit ratio $m_s(z) = \frac{a^*_s(z)}{\pi_s(z)}$, $s = \{A, T\}$, is a measure of the incentiveto save at each productivity level.[45]

Let $M_s = \frac{\int a^*_s(z)\mu(dz)}{\int \pi_s(z)\mu(dz)}$, $s = \{A, T\}$ be the aggregate target-wealth-profit ratio. In the calibration exercise, $M_A = 11.9$ and $M_T = 12.3$. Define $share^\pi_s(z) = \frac{\pi_s(z)}{\int \pi_s(z)\mu(dz)}$, $s = \{A, T\}$. I obtain

$$M_s = \frac{\int a^*_s(z)\mu(dz)}{\int \pi_s(z)\mu(dz)} = \int \frac{\pi_s(z)}{\int \pi_s(z)\mu(dz)} \cdot \frac{a^*_s(z)}{\pi_s(z)}\mu(dz) = \int share^\pi_s(z)m_s(z)\mu(dz) \tag{F.5}$$

[3]CRRA utility guarantees that $a^*_s(z)$ is bounded for all z (Krueger, 2012). Numerically, $a^*_s(z)$ is obtained by starting at $a = 0$ and iterating on the asset policy function $a'_s(a,z)$ until $a'_s(a,z) = a$.

[4]The actual wealth-to-profit ratio at a given z is not a good measure of incentive to save for entrepreneurs. For example, if an entrepreneur receives a low productivity draw after a long series of high productivity draws, the wealth-to-profit ratio would be very high, even though this entrepreneur would be actively dis-saving at the low z state. The ratio between aggregate target wealth and actual aggregate wealth is 1.09 and 1.08 under Autarky and Trade, respectively.

[5]I conduct an similar decomposition exercise based on the target-wealth-to-income ratio $\frac{a^*_s(z)}{y_s(z)}$, $s = \{A, T\}$, where $y_s(z)$ is the average total income of entrepreneurs with productivity z. This alternative approach produces similar results. I present the results based on $\pi_s(z)$ because some interest income in $y_s(z)$ is derived from wealth accumulated under different values of z.

Following Olley and Pakes (1996), I obtain the following decomposition formula:

$$
\begin{aligned}
M_T - M_A = & \int \Big(share_T^{\pi}(z) - share_A^{\pi}(z) \Big) \cdot m_A(z) \mu(dz) \\
& + \int share_A^{\pi}(z) \cdot \Big(m_T(z) - m_A(z) \Big) \mu(dz) \\
& + \int \Big(share_T^{\pi}(z) - share_A^{\pi} \Big) \cdot \Big(m_T(z) - m_A(z) \Big) \mu(dz)
\end{aligned}
\tag{F.6}
$$

The first term Equation (F.6) is the "between" change, which is the change in M if we fix $m_s(z)$ at its level under Autarky but change the profit shares to their levels under Trade. The second term is the "within" change, which is the change in M if we keep the profit shares at their levels under Autarky, but change $m_s(z)$ to its level under Trade. The last term is a co-variance term relating changes in profit shares to changes in the target-wealth-to-profit ratio. In my baseline calibration, the "between" component, the "within" component and the covariance term account for 152%, -46% and 6% of the change in the aggregate target-wealth-to-profit ratio between Autarky and Trade, respectively.

G Models with Occupational Choice

The Occupation Model

There are two symmetric countries. In each country, there is a unit measure of infinitely-lived individuals, who are heterogeneous in their wealth a. Each individual is endowed with one unit of productive labor, which is differentiated by the quality of entrepreneurial idea z. Individuals have the following utility function:

$$
U(c) = E\Big(\sum_{t=0}^{\infty} \beta^t \frac{{c_t}^{1-\lambda}}{1-\lambda} \Big), \tag{G.7}
$$

In each period, individuals choose to work for a wage w, or work as a producer of differentiated goods. Individuals have to pay a fixed cost f_D, denominated in units of labor, to operate a firm each period. An individual with an entrepreneurial idea of quality $z(i)$ can produce a variety i of differentiated goods according to

$$
q(i) = z(i) k^{\alpha} l^{1-\alpha}. \tag{G.8}
$$

The quality of an entrepreneurial idea z is drawn from a time-invariant cumulative distribution function (CDF) $\mu(z)$. Entrepreneurial ideas expire with a probability of $(1-\gamma)$ in each period. In the case of expiration of ideas, a new value of z is drawn from the CDF $\mu(z)$. I denote an individual's occupational choice as $o \in \{W, D, X\}$, where W, D and X correspond to workers, domestic producers and exporters, respectively. The specification of the final good market, the differentiated goods market, international trade, and the capital rental market are the same as in the full model.

A stationary competitive equilibrium with international trade is similar to the definition in Section 3.3, with the key modifications described below.

1. Given aggregate variables w, r^K, r, P, R, and the corresponding variables in the foreign country, individual policy functions, $c(a,z)$, $a'(a,z)$, $o(z)$, $l(z)$, $k(z)$, $q^D(z)$ and $q^X(z)$, solve an individual's optimization problem. The additional policy function $o(z) \in \{W, D, X\}$ is the occupational choice function of an individual, which is characterized by

$$\max\{w, \pi^D(z), \pi^X(z)\}.$$

2. The labor market clears.

$$\begin{aligned} \int_{o(z)=W} \int_a G(da, dz) \quad &= \quad \int_z \int_a l(z) G(da, dz) + \int_{o(z)\in\{D,X\}} \int_a G(da, dz) \cdot f_D \\ &+ \quad \int_{o(z)=X} \int_a G(da, dz) \cdot f_X \end{aligned}$$

The integral on the left hand side is taken over all workers $\{(a,z)| \, o(z) = W\}$, and gives the total labor supply. The first integral on the right-hand side is taken with respect to the entire population while the next two integrals gives total labor used as fixed costs of operation and fixed costs of exporting, respectively. Note that $l(z) = 0$ if $o(z) = W$, since workers have zero demand for labor.

3. The market for the final good clears.

$$\int_z \int_a c(a,z) G(da, dz) + \delta \cdot K \quad = \quad Y$$

The first integral on the left-hand side is taken with respect to the whole population. This term is the total consumption in the economy. The other terms on the left-hand side give the total depreciation of capital, the total fixed cost of operation and the total fixed cost of exporting, respectively. Finally, Y is the total output of the final good.

4. The joint distribution of wealth a and entrepreneurial ideas z is a fixed point of the equilibrium mapping

$$G(a,z) = \quad \gamma \textstyle\int_{\tilde{z}\leq z} \int_{a'(\tilde{a},\tilde{z})\leq a} G(d\tilde{a}, d\tilde{z}) + \quad (1-\gamma)\mu(z) \displaystyle\int_{\tilde{z}} \int_{a'(\tilde{a},\tilde{z})\leq a} G(d\tilde{a}, d\tilde{z}) \qquad \text{(G.9)}$$

For any point (a,z), the CDF at this point (LHS) should be equal to the CDF at the same point next period (RHS).

The Occupation CM Model

There is one representative agent in each country. The representative agent of a country owns a unit measure of productive labor. As in the benchmark model, productive labor is differentiated by the quality of the entrepreneurial idea z, which is drawn from the CDF $\mu(z)$. The representative agent maximizes

$$\max_{c_t, a_t} \sum_{t=0}^{\infty} \beta^t \frac{{c_t}^{1-\lambda}}{1-\lambda}, \tag{G.10}$$

subject to the dynamic budget constraint

$$c_t + a_{t+1} = \int \max\{w, \pi^D(z), \pi^X(z)\} d\mu(z) + (1+r)a_t. \tag{G.11}$$

The final good sector, the differentiated goods sector and the capital rental market are the same as before. Each unit of productive labor makes production decisions independently to maximize income. A stationary competitive equilibrium with international trade is defined accordingly.

Table G.1: Calibration of Models with Occupational Choice

Panel A: Parameters Taken from Prior Literature					
		"Occupation"		"Occupation CM"	
Parameter	Symbol	Value		Value	
Coefficient of Risk Aversion	λ	1.500		-	
Share of Capital in Production	α	0.333		0.333	
Capital Depreciation Rate	δ	0.060		0.060	
Elasticity of Substitution	σ	5.000		5.000	
Persistence of Firm Productivity	γ	0.814		-	
Shape Parameter of Sales Distribution	ζ	1.500		1.500	
Panel B: Parameters Calibrated to Match Data Moments					
		"Occupation"		"Occupation CM"	
Target Moment	US Data	Parameter	Model	Parameter	Model
Interest Rate	3.00%	$\beta = 0.952$	2.97%	$\beta = 0.971$	2.97%
Export/GDP Ratio	7.00%	$f_{EX} = 1.700$	7.20%	$f_{EX} = 1.700$	7.20%
Export to Sales Ratio	14.00%	$\tau = 1.57$	14.00%	$\tau = 1.57$	14.00%
Percentage of Entrepreneurs in Population	4.50%	$f_D = 1.270$	4.51%	$f_D = 1.270$	4.51%

"Occupation" refers to a model with occupational choice while "Occupation CM" refers to its associated complete markets benchmark.

I calibrate the Occupation and the Occupation CM models following a strategy similar to that in Section 4.1. The additional parameter to be calibrated is f_D, the fixed cost of operating a firm. I target this parameter to match the share of entrepreneurs in the population. According to 2004 data from the Bureau of Labor Statistics, the total number of firms with payroll in the US is 5.9 million while total nonfarm payroll employment is 132 million. I take the ratio of these two numbers (4.5%) as the targeted share of entrepreneurs in the population. Table G.1 provides the details on the calibration of other parameters.

Table G.2 summarizes the gains from trade from both the Occupation and Occupation CM models. Moving from Autarky to Trade, the TFP increase in the Occupation model (1.24%) is larger than in the Occupation CM model (1.02%). In the Occupation model, the change in the

Table G.2: Gains from Trade in Models with Occupational Choice

	"Occupation"	"Occupation CM"
The Effect on Aggregate Output		
TFP	1.24%	1.02%
Capital	3.78%	1.54%
Output	2.50%	1.54%
The Effect on Consumption and Welfare		
Wage	2.44%	1.44%
Welfare	1.71%	1.54%
Workers Consumption	2.68%	-
Entrepreneurial Consumption	-1.56%	-

The "Gains from Trade" numbers are the percentage differences in the relevant measure between "Autarky" and "Trade", where "Autarky" refers to the economy in stationary equilibrium when the variable cost of trade is set to infinity and "Trade" refers to the economy in stationary equilibrium calibrated to the observed level of trade in the US.

equilibrium interest rate affects the occupational choice of individuals by affecting the cost of capital and the profitability of operating a firm. As in Section 4.5, the increase in capital stock is much larger in the Occupation model (3.78%) than in the Occupation CM model (1.54%). The difference in capital accumulation again contributes to differences in gains from trade in terms of output, real wages and welfare, where the welfare measure is the certainty-equivalent consumption of a randomly-chosen individual in the economy. Lastly, moving from Autarky to Trade in the Occupation model, the total consumption by workers each period increases by 2.68% while the total consumption by entrepreneurs decreases by 1.56%. I do not provide these numbers for the Occupation CM model because incomes from all units of productive labor are pooled together in that model. The results in Table 8 and Table 9 in Section 4 are robust to the introduction of occupational choice into the model, although it is no longer possible to examine the welfare of workers and entrepreneurs separately.

H Further Trade Liberalizations

In this paper, I focus on the comparison between Autarky and an economy calibrated to match the observed level of trade in the US. The comparison reveals the realized gains from trade, which are of much interest in the trade literature. I consider an additional policy experiments. I lower the variable trade cost τ to double the import penetration ratio from 7.0% to 14.0%. This results in a value of τ of 1.38. I refer to this counterfactual economy as "More Trade"($\tau = \tau_{MT}$). As before, I also compute the equilibrium for the complete markets benchmark. Table H.3 presents the results on inequality, while Table H.4 presents the results on output and welfare gains from trade. The supply-side channel of capital adjustment appears to be more important when countries are initially less open to trade. As shown in Table H.3, the effect of further trade liberalizations on the income shares of the most productive entrepreneurs is small, and there is a very little change in wealth inequality. In the calibration, the fixed cost of exporting is relatively small, and further trade liberalization results in export entry by less productive firms. This quickly drives up the

Table H.3: Further Trade Liberalization: Results on Inequality

Panel A. Income Inequality among Entrepreneurs			
	Autarky	Trade	More Trade
Share of Total Entrepreneurial Income			
1st Quintile	5.5%	5.2%	5.0%
2nd Quintile	7.6%	7.2%	6.8%
3rd Quintile	10.5%	10.0%	9.6%
4th Quintile	16.1%	15.4%	15.0%
5th Quintile	60.3%	62.2%	63.6%
Top 10%	47.5%	49.8%	51.1%
Top 5%	37.3%	39.5%	40.5%
Gini Coefficient	0.536	0.557	0.572
Panel B. Wealth Inequality among Entrepreneurs			
	Autarky	Trade	More Trade
Share of Total Wealth			
1st Quintile	0.0%	0.0%	0.0%
2nd Quintile	1.2%	1.0%	1.0%
3rd Quintile	5.2%	4.7%	4.8%
4th Quintile	13.1%	12.5%	12.7%
5th Quintile	80.4%	81.7%	81.5%
Top 10%	66.2%	67.8%	67.6%
Top 5%	53.9%	55.5%	55.5%
Gini Coefficient	0.786	0.798	0.797

"Autarky" refers to the economy in stationary equilibrium with a variable trade cost of infinity; "Trade" refers to the economy in stationary equilibrium calibrated to the observed level of trade in the US; "More Trade" refers to the economy in stationary equilibrium in which the trade cost is calibrated to match an import penetration ratio of 14.0%.

Table H.4: Further Trade Liberalizations: Results on Output and Welfare

	Changes in Output and Welfare	
	(1)	(2)
Model	Full Model	CM Benchmark
Production		
TFP	1.44%	1.40%
Capital	3.55%	2.12%
Output	2.62%	2.11%
Consumption of workers		
Wage	3.51%	3.03%
Entrepreneurial Consumption		
Aggregate	0.33%	0.37%
Certainty-Equiv.	-2.34%	-

"Full Model" refers to the model described in Section 3.1; "CM Benchmark" refers to the complete markets benchmark described in Section 3.4. The numbers give the percentage differences in the relevant measure between economies with different variable trade costs. "Trade" refers to the economy in stationary equilibrium calibrated to the observed level of trade in the US; "More Trade" refers to the economy in stationary equilibrium in which the trade cost is calibrated to match an import penetration ratio of 14.0%.

equilibrium wage, and limits the increase in top income inequality. As a result, the difference in the response in capital between the full model and the CM benchmark is smaller in Table H.4 than in Table 8.

I Robustness and Extensions of the Calibration Exercise

In this section, I examine the robustness of the results from the baseline calibration of the full model. The results are robust to alternative specifications of the fixed cost of exporting or borrowing limits.

I.1 Sunk Cost

I first examine an alternative assumption on the fixed cost of exporting. In addition to the per-period fixed cost of exporting f_X, I assume that firms which did not export the previous period have to pay a sunk cost of exporting of f_{sunk} units of labor. As a result, the previous export status of the firm is a state variable. Table I.5 provides the details of the calibration. For simplicity, I set $f_{sunk} = 4 \cdot f_X$ in the calibration. The results are reported in Column (1) in Table I.6. The results can be compared to Columns (4) and (5), which reproduce results from the full model and from the CM benchmark, respectively. A comparison of Columns (1) and (4) indicates that the quantitative results are broadly robust to the alternative specification of the fixed cost of exporting.

Table I.5: Calibration: Extensions and Robustness

Panel A: Parameters Taken from Prior Literature						
		(1)	(2)	(3)	(5)	(6)
Model		Sunk	NBL	K-Friction	Full	CM
Parameter	Symbol	Value	Value	Value	Value	Value
Coefficient of Risk Aversion	λ	1.500	1.500	1.500	1.500	1.500
Share of Capital in Production	α	0.333	0.333	0.333	0.333	0.333
Capital Depreciation Rate	δ	0.060	0.060	0.060	0.060	0.060
Elasticity of Substitution	σ	5.000	5.000	5.000	5.000	5.000
Persistence of Firm Productivity	γ	0.814	0.814	0.814	0.814	0.814
Shape Parameter of Sales Distribution	ζ	1.500	1.500	1.500	1.500	1.500
Panel B: Parameter Calibrated to the Model						
		Sunk	NBL	K-Friction	Full Model	CM
Target Moment	Data	Parameter	Parameter	Parameter	Parameter	Parameter
Interest Rate	3.00%	$\beta = 0.952$	$\beta = 0.892$	$\beta = 0.952^{*}$	$\beta = 0.952$	$\beta = 0.971$
Import Penetration Ratio	7.00%	$f_{EX} = 0.051$	$f_{EX} = 0.091$	$f_{EX} = 0.058$	$f_{EX} = 0.088$	$f_{EX} = 0.088$
		$f_{sunk} = 0.202$	-	-	-	-
Export to Sales Ratio	14.00%	$\tau = 1.57$	$\tau = 1.57$	$\tau = 1.57$	$\tau = 1.57$	$\tau = 1.57$
Credit/GDP Ratio (Counter-factual)	60.00%	-	-	$\phi = 0.23$	-	-

[a] β is taken from the full model. It is not re-calibrated to match the interest rate.
"Sunk" refers to a modification of the full model with sunk cost of exporting. "NBL" refers to a version of the model where there is a "Natural Borrowing Limit". "K-Frictions" refers to a version of the model with financial frictions on the production side.

Table I.6: Robustness of Baseline Results

Model	(1) Sunk	(2) NBL	(3) K-Frictions	(5) Full Model	(6) CM Benchmark
TFP	1.25%	1.22%	0.82%	1.23%	1.23%
Capital	4.18%	3.37%	2.42%	3.95%	1.85%
Output	2.64%	2.35%	1.63%	2.55%	1.85%
Consumption of Workers					
Wage	3.48%	3.26%	2.99%	3.48%	2.78%
Entrepreneurial Consumption					
Aggregate	0.22%	0.02%	-1.15%	0.02%	0.14%
Certainty-Equivalent	-3.88%	-2.77%	-2.92%	-3.85%	-

"Sunk" refers to a modification of the full model with a sunk cost of exporting. "NBL" refers to a version of the model with a "Natural Borrowing Limit". "K-Frictions" refers to a version of the model with financial frictions on the production side. Columns (4) and (5) reproduce key results from Tables 8 and 9.

I.2 Natural Borrowing Limits

I have assumed so far that entrepreneurs cannot borrow ($a \geq 0$). To examine the role of the zero-borrowing assumption, I study a version of the model with a natural borrowing limit. Specifically, I assume

$$a > -\frac{\pi^D(z_{min})}{r}. \tag{I.12}$$

Inequality (I.12) requires an entrepreneur to be able to keep up with the interest payment on her loans while maintaining positive consumption, even if the entrepreneur receives the lowest possible productivity z_{min} forever. Table I.5 provides the details of the calibration. As reported in Column (2) of Table I.6, while the specification of a natural borrowing limit reduces the contribution of capital accumulation to output gains from trade, there is still a large increase of the capital stock of 3.37% when moving from Autarky to Trade.

I.3 Financial Frictions for Production

In this alternative, I assume that capital rental by firms is limited by imperfect enforceability of contracts. Entrepreneurs can default on their contracts after production has taken place. In case of default, entrepreneurs can keep a fraction $(1-\phi)$ of capital and revenue net of labor costs and export fixed costs, but they lose their financial assets deposited with the financial intermediary. In the following period, entrepreneurs regain access to financial markets despite the history of default. The parameter ϕ, $0 \leq \phi \leq 1$, indexes financial development of an economy.

The capital input k for a domestic producer with wealth a and productivity z must satisfy

$$\max_l \{R^{\frac{1}{\sigma}} q^D(z)^{1-\frac{1}{\sigma}} - w \cdot l\} - r^K k + (1+r)a \geq (1-\phi)\left\{\max_l \{R^{\frac{1}{\sigma}} q^D(z)^{1-\frac{1}{\sigma}} - w \cdot l\} + (1-\delta)\cdot k\right\} \tag{I.13}$$

where $q^D(z) = zk^{\alpha}l^{1-\alpha}$. Equation (I.13) states that a non-exporter must end up with more resources by fulfilling credit and rental obligations (left-hand side) than by defaulting (right-hand side). Equation (I.13) can be reduced to

$$\phi \cdot \max_{l} \left\{ R^{\frac{1}{\sigma}} \left(q^D(z) \right)^{1-\frac{1}{\sigma}} - w \cdot l \right\} - k \cdot \left[r^K + (1-\phi)(1-\delta) \right] + (1+r)a \geq 0, \tag{I.14}$$

which implies a capital rental limit $\bar{k}^d(a, z; \phi)$ for a non-exporter with state (a, z). Analogously,

$$\phi \cdot \max_{l} \left\{ R^{\frac{1}{\sigma}} q^D(z)^{1-\frac{1}{\sigma}} + R^{*\frac{1}{\sigma}} q^X(z)^{1-\frac{1}{\sigma}} - w \cdot l - w \cdot f_X \right\} - k \cdot \left[r^K + (1-\phi)(1-\delta) \right] + (1+r)a \geq 0, \tag{I.15}$$

which implies a capital rental limit $\bar{k}^X(a, z; \phi)$ for exporters. Entrepreneurs hire capital subject to Equations (I.14) and Equation (I.15), and there is no default in equilibrium.

The other features of the model are the same as the full model. With the financial constraints, the production policy functions are now written as $q^D(a, z)$, $q^X(a, z)$, $k(a, z)$, $l(a, z)$ and $e(a, z)$. The stationary equilibrium is defined analogously. The full model in this paper corresponds to the special case where $\phi = 1$.

I calibrate the additional parameter ϕ to match the total private credit to GDP ratio. In the calibration of the full model ($\phi = 1$), the credit to GDP ratio is 189%, compared to the figure of 162% for the US in 2000. Keeping the other parameters constant, I calibrate the financial development parameter ϕ to match a Credit/GDP ratio of 60%, which is the level of financial development studied in Buera and Shin (2013), and change the fixed cost of exporting f_X to maintain an import penetration ratio of 7%. The introduction of the enforcement constraint decreases the demand for capital and the equilibrium interest rate. The strategy of calibrating ϕ after calibrating other parameters to the US benchmark follows Buera and Shin (2013). Lastly, I increase the variable cost τ to infinity to obtain the equilibrium under Autarky. The details of the calibration are presented in Column (3) of Table I.5 while the results are presented in Table I.6. Moving from Autarky to Trade, TFP increases by 0.82%, smaller than the increase of 1.23% in the full model. The capital stock increases by 2.42%, which is larger than the increase in the CM benchmark, when we take into account the smaller increase in TFP in the K-Frictions model. Therefore, the mechanism emphasized in this paper is robust to the introduction of financial frictions on the production side.

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