The Alibaba Effect: Spatial Consumption Inequality and Welfare Gains from E-commerce
Online Appendix: An Alternative Model with Variable Markups

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1 Model

In this section, we develop a model of monopolistic competition with variable markups, by extending the model in Simonovska (2015) to incorporate both online and offline trade.

1.1 Environment

There are N regions in the economy, indexed by $i, j \in \{1, 2, ..., N\}$. Each region corresponds to a prefecture city in the data. Region i is endowed with a number of workers L_i, each with one unit of labor and supplies their labor inelastically. As in the baseline model, there are two sectors, tradable and non-tradable, indexed by T and NT, respectively. The preference of the representative consumer in region j is given by:

$$U_j = \prod_{h \in \{T, NT\}} (u_j^h)^{\beta_j^h},$$

in which u_j^h denotes the sub-utility from consuming goods in sector h, defined below, and β_j^h is the share of expenditure on sector h. The sub-utility in $h \in \{T, NT\}$ is given by:

$$u_j^h = \int_{\omega \in \Omega_j^h} log(q_j(\omega) + \bar{q}) d\omega, \quad (1)$$

in which Ω_j^h is the set of goods from sector h that are available in region j. In equilibrium, it will be determined endogenously by firms' decisions to sell to other cities. Different from the CES utility in the baseline model, the utility in Equation 1 does not generate infinite marginal utility, even when the quantity is zero, due to the shifter $\bar{q}$ in the utility function. With this modification, the elasticity of demand will no longer be constant, and more productive firms will charge higher markups.

Consider the representative household from region j. Her consumption optimization problem given total expenditures Y_j implies that $y_j^h = \beta_j^h Y_j$ is spent on sector h. In the following, we assume that trade costs in NT are infinite, and proceed as if it was tradable. Without confusion we will omit the sector superscript h from all the variables in our discussion below.

Different from in the benchmark model, in this setup, each firm produces one single variety, indexed by ω. The demand for a variety of good ω is given by

$$q_i(\omega) = \frac{y_j + \bar{q}P_j}{N_j p_j(w)} - \bar{q}, \tag{2}$$

where $P_j = \int_{\omega \in \Omega_j} p_j(\omega) d\omega$ is the sum of prices of all firms selling in market j, and $N_j = \sum_{i=1}^{N} N_{ij}$ is the total number of varieties from all over the countries available in region j, with N_{ij} denoting the total number of varieties sold from i to j. As Equation 2 shows, the demand $q_j(\omega)$ is not necessarily positive. Indeed, when $p_j(\omega) = \frac{y_j + \bar{q}P_j}{N_j \bar{q}}$, the demand is exactly zero. It follows that the demand will be zero for firms whose marginal cost of serving region j is exactly $\frac{y_j + \bar{q}P_j}{N_j \bar{q}}$. We use $\bar{c}_j$, given by the following expression, to denote this cutoff

$$\bar{c}_j = \frac{y_j + \bar{q}P_j}{N_j \bar{q}}.$$

As one might imagine, in a world without fixed costs, firms who are able to sell to market j with a marginal cost of $\bar{c}_j$ will only be able to charge their marginal cost. Firms who are able to deliver their products to market j at a cost lower than $\bar{c}_j$ will charge some markups. We now turn to the firm side.

1.2 Firms' Problem

There are two types of firms in the economy, traditional (denoted by superscript 'P') and e-commerce (denoted by 'E') firms. Each firm of both types produces one differentiated variety. While traditional firms rely on brick-and-mortar stores when selling to other cities, e-commerce firms can sell directly online without incurring any destination-specific costs. Firms set prices and compete monopolistically.

We assume that region i is endowed with an exogenous measure M_i^P of traditional firms. These firms have their productivity drawn from $F_i(\phi)$ distribution. We assume that the fixed cost to open up a brick-and-mortar store in region j is f^P units of labor in region j. There is an iceberg cost for a firm to ship goods to its physical stores. We denote the iceberg shipping cost between i and j using τ_{ij}^P.

In contrast to traditional firms, e-commerce firms do not have to incur any destination-specific costs. Nevertheless, these firms need to incur a variable shipping cost when they want to sell to customers in other cities. We use τ_{ij}^E to denote the iceberg shipping cost associated with e-commerce companies. The measure of e-commerce firms, M_i^E, is determined by the entry decision of these firms. To enter, potential entrants pay a fixed cost of F_{Entry} in terms of region i labor, after which they draw a productivity value ϕ from $F_i(\phi)$. We assume that $F_i(\phi)$ follows a Pareto distribution, $F_i(\phi) = 1 - (\frac{\phi}{\underline{\phi_i}})^{-\alpha}$, where α is the shape parameter and $\underline{\phi_i}$ is the technology level.

We now discuss firms' entry and pricing decisions. Consider a firm from region i selling to region j. The variable profit from region j is given by:

$$\pi_j(\omega) = [p(\omega) - c(\omega)][\frac{y_j + \bar{q}P_j}{N_j p(\omega)} - \bar{q}],$$

where $p(\omega)$ is the price, and $c(\omega)$ is the cost of producing and delivering one unit of output to consumers in region j. Solving this optimization problem, we have the following pricing equation:

$$p(\omega) = [c(\omega)\bar{c}_j]^{\frac{1}{2}}. \tag{3}$$

Clearly, from this equation, for firms to be able to charge any markup, i.e., for $\frac{p(\omega)}{c(\omega)} \geq 1$ to hold, we must have $c(\omega) < \bar{c}_j$. As a result, only firms with marginal cost of production (inclusive of shipping ocsts) below $\bar{c}_j$ would be able to make a profit in region j. Substituting the pricing equation into the expression for variable profit from region j, we have:

$$\pi_j(c) = \bar{q}[\bar{c}_j^{\frac{1}{2}} - c^{\frac{1}{2}}]^2, c \leq \bar{c}_j. \tag{4}$$

Equation 4 is a quadratic function centered around $\bar{c}_j$. When $0 < c < \bar{c}_j$, $\pi(c)$ decreases monotonically with c. The profit is maximized when c approaches 0, with a maximum value of $\bar{q}\bar{c}_j$. The bounded maximum profit also implies it is possible that for destination cities, offline firms will not choose to enter no matter how productive they are—the fixed cost might be larger than the maximum possible profit. On the other hand, if some traditional firms from region i are able to recoup their upfront investment, $w_j f^P$, where w_j is the wage in region j, they would sell to consumers in region j. In this case, there will be a sorting equilibrium a la Melitz, i.e., there exists a cutoff, $\hat{\phi}_{ij}^P$, so that firms from region i with productivity above this cutoff sells to region j. Letting $\bar{q}[\bar{c}_j^{\frac{1}{2}} - c^{\frac{1}{2}}]^2 = f^P w_j$, the cutoff marginal cost should be $c = [\bar{c}_j^{\frac{1}{2}} - (\frac{f^P w_j}{\bar{q}})^{\frac{1}{2}}]^2$. Noting that $c = \frac{w_i \tau_{ij}^P}{\phi}$, we have

$$\hat{\phi}_{ij}^P = \frac{w_i \tau_{ij}^P}{[\bar{c}_j^{\frac{1}{2}} - (\frac{f^P w_j}{\bar{q}})^{\frac{1}{2}}]^2}, \quad \bar{c}_j^{\frac{1}{2}} > (\frac{f^P w_j}{\bar{q}})^{\frac{1}{2}}. \tag{5}$$

In the event that $\bar{c}_j^{\frac{1}{2}} < (\frac{f^P w_j}{\bar{q}})^{\frac{1}{2}}$, even the most productive firm does not find it profitable to enter and there will be no entry for these region pair among all traditional firms.

The sales from i to j by an offline firm with productivity $\phi, \phi \geq \hat{\phi}_{ij}^P$, is given by:

$$s_{ij}^P(\phi) = \bar{q}[\bar{c}_j - \bar{c}_j^{\frac{1}{2}} c_{ij}^{P\,\frac{1}{2}}(\phi)],$$

in which $c_{ij}^P(\phi)$ is the cost for a offline firm to sell to j. The labor demanded for these sales is given by:

$$l_{ij}^{P}(\phi) = \frac{\tau_{ij}^{P}\bar{q}(\bar{c}_j^{\frac{1}{2}}(\frac{w_i\tau_{ij}^{P}}{\phi})^{-\frac{1}{2}} - 1)}{\phi}.$$

Firm ϕ make the following variable profit from these sales:

$$\pi_{ij}^{P}(\phi) = \bar{q}[\bar{c}_j^{\frac{1}{2}} - (\frac{w_i\tau_{ij}^{P}}{\phi})^{\frac{1}{2}}]^2.$$

For e-commerce firms, since there are no fixed costs, the cutoff is given by $\bar{c}_j$, i.e.,

$$\hat{\phi}_{ij}^{E} = \frac{w_i\tau_{ij}^{E}}{\bar{c}_j}. \tag{6}$$

We can similarly derive the sales, profit, and labor demand from variable production for e-commerce firms:

$$s_{ij}^{E}(\phi) = \bar{q}[\bar{c}_j - \bar{c}_j^{\frac{1}{2}}c_{ij}^{E\,\frac{1}{2}}(\phi)],$$

$$l_{ij}^{E}(\phi) = \frac{q(\phi)}{\phi/\tau_{ij}^{E}} = \frac{\tau_{ij}^{E}\bar{q}(\bar{c}_j^{\frac{1}{2}\tau_{ij}^{E}}(\frac{w_i\tau_{ij}^{E}}{\phi})^{-\frac{1}{2}} - 1)}{\phi},$$

and

$$\pi_{ij}^{E}(\phi) = \bar{q}[\bar{c}_j^{\frac{1}{2}} - (\frac{w_i\tau_{ij}^{E}}{\phi})^{\frac{1}{2}}]^2.$$

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To see how this model introduce pro-competitive effects, consider a change to $\bar{c}_j = \frac{y_j + \bar{q}P_j}{N_j\bar{q}} = \frac{y_j}{N_j\bar{q}} + \frac{P_j}{N_j}$. As the number of varieties available increases (while keeping the average price $\frac{P_j}{N_j}$ unaltered), the first order impact will be a decrease in the first term. This leads to a decrease in $\bar{c}_j$, which lower the markup $\frac{p}{c} = (\frac{\bar{c}_j}{c})^{\frac{1}{2}}$. This will further decrease $\frac{P_j}{N_j}$, leading to additional effects.

1.3 Aggregation

We can derive the equations that characterize the general equilibrium of the model based on the above firm-level variables. Under Pareto distribution for firms' productivity, $F_i(\phi)$, these firm-level outcomes can be easily aggregated.

The total sales by offline firms from i to j are given by

$$S_{ij}^{P} = M_i^{P}\int_{\hat{\phi}_{ij}^{P}}^{\infty} s_{ij}^{P}(\phi)dF_i(\phi)$$

The total sales by online firms from i to j are

$$S_{ij}^E = M_i^E \int_{\hat{\phi}_{ij}^E}^{\infty} s_{ij}^E(\phi) dF_i(\phi)$$

The total profit made by offline firms in i from selling to j, netting out of fixed cost, is:

$$\Pi_{ij}^P = M_i^P \int_{\hat{\phi}_{ij}^P}^{\infty} [\pi_{ij}^P(\phi) - f^P w_j] dF_i(\phi)$$

Similarly, we can derive the total profit made by online firms in i from selling to j. Here we have no fixed costs:

$$\Pi_{ij}^E = M_i^E \int_{\hat{\phi}_{ij}^E}^{\infty} \pi_{ij}^E(\phi) dF_i(\phi)$$

The total number of varieties available in region i is:

$$N^i = \sum_{j=1}^{N} \{M_j^P[1 - F_j(\hat{\phi}_{ji}^P)] + M_j^E[1 - F_j(\hat{\phi}_{ji}^E)]\}$$

1.4 The Equilibrium

The equilibrium of this economy is defined as a set of wages w_i, prices P_i; number of varieties M_j^h; and decision rules $\hat{\phi}_{ij}^E$ and $\hat{\phi}_{ij}^P$, such that that given exogenous parameters, the following conditions are satisfied:

1. Consumers maximize utility. This gives rise to the demand function given by Equation 2.

2. Firms maximize profits according the markup rule, given by Equation 3, and cost cutoffs for market entry, given by Equations 5 and 6.

3. Total consumption expenditure of a region is equal to the total income. In this economy, total income is simply the total wage income and total profit by traditional firms, since the e-commerce firms are making zero profit. Total expenditure is given by:

$$Y_i = w_i L_i + \sum_{j=1}^{N} \Pi_{ij}^P. \tag{7}$$

4. The equilibrium price in both sectors of any region is consistent with the distribution of wages, prices and available varieties. The price index in region j is:

$$P_j = \sum_{i=1}^{N} [M_i^P \int_{\hat{\phi}_{ij}^P}^{\infty} (\bar{c}_j \frac{w_i \tau_{ij}^P}{\phi})^{\frac{1}{2}} dF_i(\phi) + M_i^E \int_{\hat{\phi}_{ij}^E}^{\infty} (\bar{c}_j \frac{w_i \tau_{ij}^E}{\phi})^{\frac{1}{2}} dF_i(\phi)]$$

5. Labor market clears in each region. The labor market clearing condition is given by:

$$L_i = M_i^E F_{Entry} + \sum_{j=1}^{N} \{L_{ij}^P + L_{ij}^E + M_j^P [1 - F_j(\hat{\phi}_{ji}^P)] f^P\} \tag{8}$$

where the total labor demand by offline firms for fulfilling sales from i to j, L_{ij}^P, is given by

$$L_{ij}^P = M_i^P \int_{\hat{\phi}_{ij}^P}^{\infty} l_{ij}^P(\phi) dF_i(\phi)$$

and L_{ij}^E is similarly given by

$$L_{ij}^E = M_i^E \int_{\hat{\phi}_{ij}^E}^{\infty} l_{ij}^E(\phi) dF_i(\phi)$$

6. Free Entry. The zero profit condition for e-commerce firms is

$$M_i^E w_i F_{Entry} = \sum_{j=1}^{N} \Pi_{ij}^E \tag{9}$$

1.5 Discussion of Model Assumptions

The model imposes several assumptions that deviate from the baseline model presented in the main text. We discuss the reasoning behind these choices here. First, in the current model, online and offline firms are separate entities—some firms are born as online firms, who can use the e-commerce technology, while other firms are born as offline firms, who need to open up brick-and-mortar stores before selling their products into a region. We adopt this assumption primarily for computational tractability. Specifically, in many models with variable markups, including this one, the profit a firm can earn is bounded from above (see Equation 4). This means, if we allow firms to decide whether to sell online, offline, or both, as we do in the baseline model (or in the spirit of Helpman et al., 2004), the decision to open up a physical store will not necessarily be monotonic: while some medium-productivity firms might find it profitable to open up physical stores, the most productive firms, who are already close to making the maximum profit through the online channel, will not choose to do so. This introduces difficulty in characterizing firm-level decisions analytically, so we abstract from online-offline cannibalization in this model.

Second, we assume that while the mass of online firms is determined by endogenous entry, the mass of traditional firms is determined exogenously. This is because in this model, with online and offline firms being separate entities, the relative mass of these two types of firms is not uniquely determined. For this reason, we choose to fix the mass of the traditional firms, and allow e-commerce to affect the mass of online firms endogenously. One obvious concern with this approach is that it rules out potential home-market effects, which might hurt small and remote cities. Our baseline model, however, suggests that for the main points of our paper, this effect might not be important. Indeed, in a working paper version of the main text, we use a model with exogenous entry. The results obtained in that model are similar to those from a model with free entry.

2 Quantification and Results

Table 1: Calibrated Parameters and Targets Moments in the Model with Variable Markups

Parameter	Value	Target Moment	Data	Model
Fixed Costs				
F_{Entry}	1.000	Average Firm Size	80	80
f^P	6.301	Share Of Firms Selling Offline to Another City	0.940	0.850
Offline Shipping Cost				
δ_1	-0.213	Province Absorption Ratio	0.540	0.443
δ_2	0.797	Distance Elasticity in Offline Gravity	-1.400	-1.601
δ_L	0.003	Pop Elasticity of Online Share	-0.110	-0.109
Online Shipping Cost				
γ_0	2.382	Average Online Share	0.080	0.080
γ_1	2.910	Province Online Absorption Ratio	0.145	0.144
γ_2	0.137	Distance Elasticity in Online Gravity Equation	-0.470	-0.470

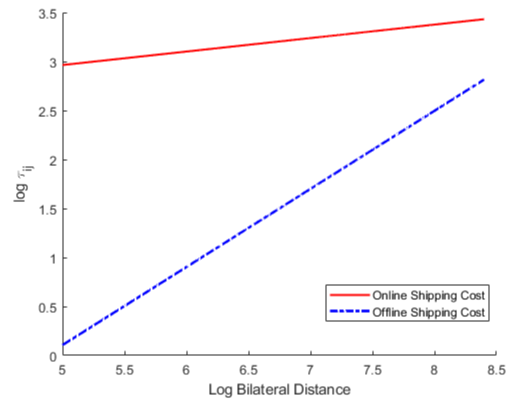
Notes: calculation based on model simulations of a subsample of 50 cities. The parameter F_{Entry} refers to entry cost for *e-commerce firms* while f^P refers to market entry cost for *traditional firms*. See Equations 17 and 18 in the main text for parameterization of shipping costs.

To take this model with variable markups to the data, we follow the quantification exercise in our baseline calibration closely. However, it turns out that the computational burdens of this model are significantly higher than the baseline. Consequently, we draw a random sample of 50 cities from the full sample and conduct the quantification exercise based on these 50 cities. Consequently, we view the quantification exercise as a numerical illustration of the robustness of our baseline results.

The model features a new parameter $\bar{q}$ in the utility function. Our assumption on utility function requires each sectoral sub-utility function to non-negative. To avoid negative values for sectoral utility, we set $\bar{q}$ to be 1. The Pareto shape parameter of productivity α under the model with variable markups is different from that in the baseline, as the mapping between productivity distribution and firm sales distribution no longer holds. We set $\alpha = 1.88$ following Jung, Simonovska and Weinberger (2015). Lastly, as in the baseline, we set β_j^{NT} to be match the expenditure share of service of each city.

To calibrate the shipping costs in the model, we follow the same procedure, including the parametrization of the shipping costs, as in the baseline. Specifically, for a parameter value, we simulate the model to calculate a set of moments. We then search over the parameter space to minimize the differences between the model moments and their data counterparts. In practice, we

Figure 1: Calibrated Offline and Online Shipping Costs



Notes: calculation based on model simulations of a sub-sample of 50 cities.

are able to match all the targeted moments in the baseline calibration well except for one, namely the elasticity of online share with respect to market potential. A possible explanation is that the sub-sampling of cities is unable to preserve the spatial configuration of the model economy with regard to market potential. Consequently, we drop this moment from the set of targeted moments. Table 1 presents the goodness of fit from our calibration. As Table 1 shows, we are able to match the targeted moments well.

Table 1 presents calibrated values of the parameters in our model. As in the baseline calibration, we compare the calibrated values of online and offline shipping costs. Figure 1 plots the log of the calibrated shipping costs τ_{ij}^{P} and τ_{ij}^{E} against the log of bilateral distance. As Figure 1 shows, the level of online shipping cost is higher than the level of offline shipping cost. Second, as in the baseline calibration, relative to offline shipping cost, online shipping cost increases more gradually as bilateral distance increases. Therefore, the calibrated shipping costs in the model with variable markups retain the salient features of the calibrated shipping costs in the baseline.

We can now study the welfare gains from e-commerce in this model. In our calibration procedure, we have computed the equilibria for two model economies, one without and one with e-commerce. We use equivalence variation as our measure of welfare gains from e-commerce. Specifically, starting from the economy without e-commerce and fixing prices and number of available varieties, we compute the income increase needed to attain the welfare level in the economy with e-commerce.[1] Table 2 presents the results. The average welfare gains from the model is 1.26%, lower than the gains of 1.6% in the baseline. As Table 2 shows, online expenditure share decreases with population size and market potential. The average welfare gains is as much as 2.10% for cites with an above median population, relative to an average of 0.43% for those below

[1]Specifically, note that $U_j = \Pi_h (u_j^h)^{\beta^h}$ and $u_j^h = \int_{\omega \in \Omega_i^h} log(\frac{y_j^h + \bar{q}P_j}{N_j p_j(w)}) d\omega$, where $y_j^h = \beta^h Y_j$. To compute the equivalence variation, we fix the prices and varieties in each cities to the level before e-commerce, and compute the change in Y_j in order to reach the utility level attained in the economy with e-commerce.

the median. and 2.03% for cities with an above median market potential, relative to an average of 0.50% for those below the median. Crucially, online expenditure share remains a good predictor of welfare gains from e-commerce in the model with variable markups. Therefore, e-commerce can significantly reduce spatial consumption inequality in this model with variable markups.

Table 2: Welfare Gains from e-Commerce in the Model with Variable Markups

Panel A	by population	
Outcome	<median	>median
Online Share	13.74%	7.55%
Welfare Gains	2.10%	0.43%
Panel B	by market potential	
Outcome	<median	>median
Online Share	15.34%	5.95%
Welfare Gains	2.03%	0.50%

Note: calculation based on model simulations of a subsample of 50 cities. 'Online share' refers to expenditure spent on the e-commerce channel as a share of total expenditure in the tradable sector. 'Welfare gains' are measured in terms of the equivalence variation, which is the real income increase needed to attain the welfare level in the economy with e-commerce while fixing prices and number of available varieties.

Due to computational burden of the model with variable markups, we have to restrict our quantification exercise to a sample of 50 cities. As a result, we view this exercise as a numerical illustration to corroborate our baseline results. We summarize our findings from the model with variable markups as follows. First, the exercise using the model with variable markups retains the key features of the calibrated shipping costs in the baseline. Second, online share is a good predictor of welfare gains from e-commerce in this model. Third, since the small and remote cities enjoy larger gains, e-commerce can significantly reduce spatial consumption inequality.

References

Helpman, Elhanan, Marc Melitz, and Stephen Yeaple, "Export Versus FDI with Heterogeneous Firms," *American Economic Review*, 2004, *94.*

Jung, Jae Wook, Ina Simonovska, and Ariel Weinberger, "Exporter Heterogeneity and Price Discrimination: A Quantitative View," 2015.

Simonovska, Ina, "Income Differences and Prices of Tradables: Insights from an Online Retailer," *Review of Economic Studies*, 2015, *82* (4), 1612–1656.